

Rent seeking games with tax evasion

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Abstract

We consider the static and dynamic models of Cournot duopoly with tax evasion. In the dynamic model we introduce the time delay and we analyze the local stability of the stationary state. There is a critical value of the delay when the Hopf bifurcation occurs.

Mathematics Subject Classification: 34K18, 47N10; *Jel Classification:* C61, C62, H26

Keywords: Delayed differential equation, Hopf bifurcation, tax evasion.

1 Introduction

During the last decades revenues from indirect tax have become increasingly important in many economies. Substantial attention has been devoted to evasion of indirect taxes. It is well known that indirect tax evasion, especially evasion of VAT, may erode a substantial part of tax revenues [2], [4], [5].

In [3] a model with tax evasion is presented. The authors consider n firms which enter the market with a homogenous good. These firms have to pay an ad valorem sales tax, but may evade a certain amount of their tax duty. The aims of the firms are to maximize their profits. The equilibrium point is determined and an economic analysis is made.

Based on [1], [3], [7], [8], [10], in our paper we present three economic models with tax evasion: the static model of Cournot duopoly with tax evasion in Section 2, the dynamic model of Cournot duopoly with tax evasion in Section 3 and the dynamic model with tax evasion and time delay in Section 4.

In Section 2, in the static model the purpose of the firms is to maximize their profits. We determine the firms' outputs and the declared revenues which maximize the profits, as well as the conditions for the model's parameters in which the maxim profits are obtained. Using Maple 11, the variables orbits are displayed.

In Section 3, the dynamic model describes the variation in time of the firms' outputs and the declared revenues. We study the local stability for the stationary state and the conditions under which it is asymptotically stable.

In Section 4, we formulate a new dynamic model, based on the model from Section 3, in which the time delay is introduced. That means, the two firms do not enter the market

at the same time. One of them is the leader firm and the other is the follower firm. The second one knows the leader's output in the previous moment $t - \tau$, $\tau \geq 0$.

Using classical methods [6], [9] we investigate the local stability of the stationary state by analyzing the corresponding transcendental characteristic equation of the linearized system. By choosing the delay as a bifurcation parameter we show that this model exhibits a limit cycle.

Finally numerical simulations, some conclusions and future research possibilities are offered.

2 The static model of Cournot duopoly with tax evasion

The static model of Cournot duopoly is described by an economic game, where two firms enter the market with a homogenous consumption product. The elements which describe the model are: the quantities which enter the market from the two firms $x_i \geq 0$, $i = \overline{1, 2}$; the declared revenues z_i , $i = \overline{1, 2}$; the inverse demand function $p : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ (p is a derivable function with $p'(x) < 0$, $\lim_{x \rightarrow a_1} p(x) = 0$, $\lim_{x \rightarrow 0} p(x) = b_1$, ($a_1 \in \mathbb{R}$, $b_1 \in \mathbb{R}$); the penalty function $F : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ (F is a derivable function with $F'(x) > 0$, $F''(x) > 0$, $F(0) = 0$); the cost functions $C_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ (C_i are derivable functions with $C'_i(x_i) > 0$, $C''_i \geq 0$, $i = \overline{1, 2}$). The government levies an ad valorem tax on each firm's sales at the rate $t_1 \in (0, 1)$ and $q \in [0, 1]$ is the probability with which the tax evasion is detected.

The true tax base of firm i is $x_i p(x_1 + x_2)$. Firm i declares $z_i \leq x_i p(x_1 + x_2)$ as tax base to the tax authority. Accordingly, evaded revenues of firm i are given by $x_i p(x_1 + x_2) - z_i$. With probability $1 - q$ tax evasion remains undetected and the tax bill of firm i amounts to $t_1 z_i$. The tax authority detects tax evasion of firm i with probability q . In case of detection, firm i has to pay taxes on the full amount of revenues, $x_i p(x_1 + x_2)$, and, in addition, a penalty $F(x_i p(x_1 + x_2) - z_i)$. The penalty is increasing and convex in evaded revenues $x_i p(x_1 + x_2) - z_i$. Moreover, it is assumed that $F(0) = 0$, namely law-abiding firms go unpunished.

The profit functions of the two firms are: $P_i : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$, $i = \overline{1, 2}$, given by:

$$P_i = P_i(x_1, x_2, z_1, z_2) = (1 - q)[x_i p(x_1 + x_2) - C_i(x_i) - t_1 z_i] + q[(1 - t_1)x_i p(x_1 + x_2) - C_i(x_i) - F(x_i p(x_1 + x_2) - z_i)]. \quad (1)$$

The first bracketed term in (1) equals the profit of firm i if evasion activities remain undetected. The second term in (1) represents the profit of firm i in case tax evasion is detected.

The firm's aim is to maximize (1) with respect to output x_i and declared revenues z_i . This aim represents a mathematical optimization problem which is given by:

$$\max_{\{x_i, z_i\}} P_i, \quad i = \overline{1, 2}. \quad (2)$$

From the hypothesis about the functions p, F, C_i , $i = \overline{1, 2}$, we have:

Proposition 1 *The solution of problem (2) is given by the solution of the following system:*

$$\begin{aligned} \frac{\partial P_i}{\partial x_i} &= [1 - qt_1 - qF'(x_i p(x_1 + x_2) - z_i)][p(x_1 + x_2) + x_i p'(x_1 + x_2)] - C_i(x_i) = 0 \\ \frac{\partial P_i}{\partial z_i} &= -(1 - q)t_1 + qF'(x_i p(x_1 + x_2) - z_i) = 0, \quad i = \overline{1, 2}. \end{aligned} \quad (3)$$

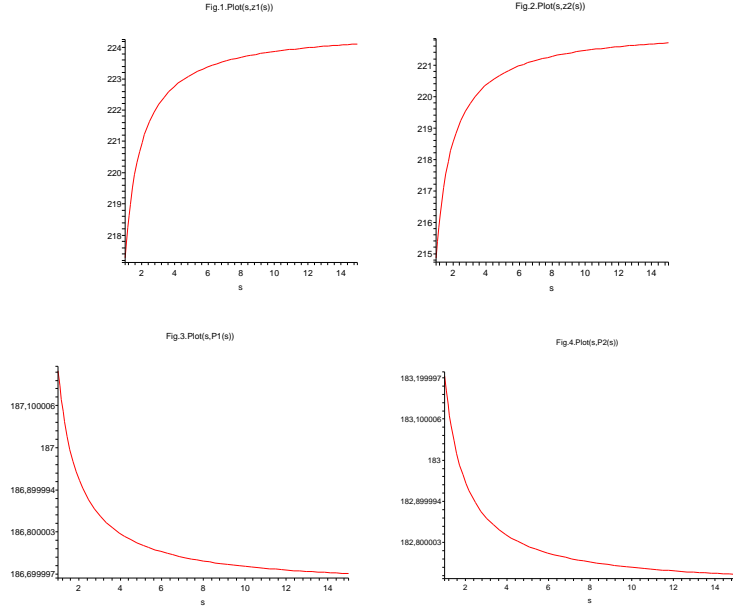
In what follows, we will consider the penalty function $F(x) = \frac{1}{2}st_1x^2$, $s \geq 1$, the cost functions $C_i(x_i) = c_ix_i$, $c_i > 0$, $i = 1, 2$ and the price function $p(x) = \frac{1}{x}$.

From (3) we can deduce:

Proposition 2 *If $\frac{1-q}{qs+q-1}c_1 \leq c_2 \leq \frac{qs+q-1}{1-q}c_1$, then the solution of system (3) is given by :*

$$\begin{aligned} x_1^* &= \frac{c_2(1-t_1)}{(c_1+c_2)^2}, & x_2^* &= \frac{c_1(1-t_1)}{(c_1+c_2)^2}, \\ z_1^* &= \frac{c_2}{c_1+c_2} - \frac{1-q}{qs}, & z_2^* &= \frac{c_1}{c_1+c_2} - \frac{1-q}{qs}. \end{aligned} \quad (4)$$

For the parameters $c_1 = 0.3$, $c_2 = 0.6$, $q = 0.12$, $t_1 = 0.16$, the variations of the variables z_1 , z_2 , and the profits P_1 , P_2 are given in the following figures:



3 The dynamic model of Cournot duopoly with tax evasion

The dynamic model describes the variation in time of output $x_i(t)$, $i = 1, 2$ taking into account the marginal profits $\frac{\partial P_i}{\partial x_i}$, $i = 1, 2$. Assume that each agent adjusts its declared revenue $z_i(t)$, $i = 1, 2$ proportionally to the marginal profits $\frac{\partial P_i}{\partial z_i}$, $i = 1, 2$. Then, the

dynamic model is given by the following differential system of equations:

$$\begin{aligned}\dot{x}_i(t) &= k_i \frac{\partial P_i}{\partial x_i} = k_i \{ [1 - qt_1 - qF'(x_i p(x_1 + x_2) - z_i)] \cdot \\ &\quad [p(x_1 + x_2) + x_i p'(x_1 + x_2)] - C_i(x_i) \}, \\ \dot{z}_i(t) &= h_i \frac{\partial P_i}{\partial z_i} = h_i [-(1 - q)t_1 + qF'(x_i p(x_1 + x_2) - z_i)], \quad i = \overline{1, 2}\end{aligned}\quad (5)$$

with the initial conditions $x_i(0) = x_{i0}$, $z_i(0) = z_{i0}$, $i = \overline{1, 2}$ and $h_i > 0$, $k_i > 0$, $i = \overline{1, 2}$.

For $F(x) = \frac{1}{2}st_1x^2$, $s \geq 1$ and $C_i(x_i) = c_i x_i$, $c_i > 0$, $i = \overline{1, 2}$ system (5) becomes :

$$\begin{aligned}\dot{x}_1(t) &= k_1 \{ [1 - qt_1 - qst_1(x_1(t)p(x_1(t) + x_2(t)) - z_1(t))] \cdot \\ &\quad \cdot [p(x_1(t) + x_2(t)) + x_1(t)p'(x_1(t) + x_2(t))] - c_1 \} \\ \dot{x}_2(t) &= k_2 \{ [1 - qt_1 - qst_1(x_2(t)p(x_1(t) + x_2(t)) - z_2(t))] \cdot \\ &\quad \cdot [p(x_1(t) + x_2(t)) + x_2(t)p'(x_1(t) + x_2(t))] - c_2 \} \\ \dot{z}_1(t) &= h_1 [-(1 - q)t_1 + qst_1(x_1(t)p(x_1(t) + x_2(t)) - z_1(t))] \\ \dot{z}_2(t) &= h_2 [-(1 - q)t_1 + qst_1(x_2(t)p(x_1(t) + x_2(t)) - z_2(t))] \\ x_i(0) &= x_{i0}, \quad z_i(0) = z_{i0}, \quad i = 1, 2.\end{aligned}\quad (6)$$

System (6) has the stationary state $(x_1^*, x_2^*, z_1^*, z_2^*)$ given by (4).

Let $u_1(t) = x_1(t) - x_1^*$, $u_2(t) = x_2(t) - x_2^*$, $u_3(t) = z_1(t) - z_1^*$, $u_4(t) = z_2(t) - z_2^*$.

By expanding (6) in a Taylor series around the stationary state $(x_1^*, x_2^*, z_1^*, z_2^*)$ and neglecting the terms of higher order than the first order, we have the following linear approximation of system (6) :

$$\begin{aligned}\dot{u}_1(t) &= k_1 \{ a_{10}u_1(t) + a_{01}u_2(t) + a_{001}u_3(t) \} \\ \dot{u}_2(t) &= k_2 \{ b_{10}u_1(t) + b_{01}u_2(t) + b_{001}u_4(t) \} \\ \dot{u}_3(t) &= h_1 \{ c_{10}u_1(t) + c_{01}u_2(t) + c_{001}u_3(t) \} \\ \dot{u}_4(t) &= h_2 \{ d_{10}u_1(t) + d_{01}u_2(t) + d_{001}u_4(t) \}\end{aligned}\quad (7)$$

where:

$$\begin{aligned}a_{10} &= -qst_1 \frac{c_1^2}{(1 - t_1)^2} + (1 - t_1) (2p'(x_1^* + x_2^*) + x_1^* p''(x_1^* + x_2^*)), \\ a_{01} &= -qst_1 x_1^* p'(x_1^* + x_2^*) \frac{c_1}{1 - t_1} + (1 - t_1) (p'(x_1^* + x_2^*) + x_1^* p''(x_1^* + x_2^*)), \\ a_{001} &= \frac{qst_1 c_1}{1 - t_1}, \quad b_{001} = \frac{qst_1 c_2}{1 - t_1}, \quad c_{001} = d_{001} = -qst_1, \quad c_{10} = \frac{qst_1 c_1}{1 - t_1}, \\ b_{10} &= -qst_1 x_2^* p'(x_1^* + x_2^*) \frac{c_2}{1 - t_1} + (1 - t_1) (p'(x_1^* + x_2^*) + x_2^* p''(x_1^* + x_2^*)), \\ b_{01} &= -qst_1 \frac{c_2^2}{(1 - t_1)^2} + (1 - t_1) (2p'(x_1^* + x_2^*) + x_2^* p''(x_1^* + x_2^*)), \\ c_{01} &= qst_1 x_1^* p'(x_1^* + x_2^*), \quad d_{10} = qst_1 x_2^* p'(x_1^* + x_2^*), \quad d_{01} = qst_1 \frac{c_2}{1 - t_1}.\end{aligned}\quad (8)$$

The characteristic equation associated to (7) is given by:

$$\lambda^4 + m_{43}\lambda^3 + m_{42}\lambda^2 + m_{41}\lambda + m_{40} = 0 \quad (9)$$

where:

$$\begin{aligned} m_{43} &= -k_1 a_{10} - k_2 b_{01} - h_1 c_{001} - h_2 d_{001} \\ m_{42} &= k_1 k_2 a_{10} b_{01} + (k_1 a_{10} + k_2 b_{01}) (h_1 c_{001} + h_2 d_{001}) - h_1 k_1 a_{001} c_{10} - \\ &\quad - k_2 h_2 b_{001} d_{01} + h_1 h_2 c_{001} d_{001} - k_1 k_2 a_{01} b_{10} \\ m_{41} &= k_1 k_2 a_{10} b_{01} (h_1 c_{001} - h_2 d_{001}) - k_2 h_1 h_2 c_{001} d_{001} b_{01} + \\ &\quad + h_1 k_1 c_{10} a_{001} (k_2 b_{01} + h_2 d_{001}) - k_1 k_2 h_2 b_{001} a_{01} d_{10} + \\ &\quad + k_1 k_2 h_2 a_{10} b_{001} d_{01} + k_2 h_1 h_2 b_{001} c_{001} d_{01} + \\ &\quad + k_1 k_2 a_{01} b_{10} (h_2 d_{001} + h_1 c_{001}) - k_1 k_2 h_1 a_{001} b_{10} c_{01} \\ m_{40} &= k_1 k_2 h_1 h_2 (a_{10} b_{01} c_{001} d_{001} - a_{001} b_{01} c_{10} d_{001} + a_{001} b_{001} c_{10} d_{01} + \\ &\quad + a_{01} b_{001} c_{001} d_{10} - a_{001} b_{001} c_{01} d_{10} - a_{10} b_{001} c_{001} d_{01} + \\ &\quad + a_{001} b_{10} c_{01} d_{001} - a_{01} b_{10} c_{001} d_{001}). \end{aligned} \quad (10)$$

A necessary and sufficient condition as equation (9) has all roots with negative real part is given by Routh-Hurwitz criterion:

$$D_1 = m_{43} > 0, D_2 = m_{43}m_{42} - m_{41} > 0, D_3 = m_{41}D_2 - m_{43}^2m_{40} > 0, D_4 = m_{41}D_3 > 0. \quad (11)$$

From the Routh-Hurwitz criterion we have:

Proposition 3 *The stationary state of system (6) is asymptotically stable if and only if conditions (11) hold.*

4 The dynamic model with tax evasion and time delay

In [7] and [8] we have studied the rent seeking games with time delay and distributed delay. In the present section we analyze the rent seeking games with tax evasion and delay. For $\tau = 0$ we obtain the model from [3]. For $\tau = 0$ and $t_1 = 0$ we obtain the model from [1]. We consider the model from section 3 where we introduce the time delay τ . We suppose the first firm is the leader and the second firm is the follower. The follower knows the quantity of the leader firm, $x_1(t - \tau)$, which entered the market at the moment $t - \tau$, $\tau > 0$.

The differential system which describes this model is given by:

$$\begin{aligned} \dot{x}_1(t) &= k_1 \{ [1 - qt_1 - qst_1 (x_1(t)p(x_1(t) + x_2(t)) - z_1(t))] \cdot \\ &\quad [p(x_1(t) + x_2(t)) + x_1(t)p'(x_1(t) + x_2(t))] - c_1 \} \\ \dot{x}_2(t) &= k_2 \{ [1 - qt_1 - qst_1 (x_2(t)p(x_1(t - \tau) + x_2(t)) - z_2(t))] \cdot \\ &\quad [p(x_1(t - \tau) + x_2(t)) + x_2(t)p'(x_1(t - \tau) + x_2(t))] - c_2 \} \\ \dot{z}_1(t) &= h_1 [-(1 - q)t_1 + qst_1 (x_1(t)p(x_1(t) + x_2(t)) - z_1(t))] \\ \dot{z}_2(t) &= h_2 [-(1 - q)t_1 + qst_1 (x_2(t)p(x_1(t) + x_2(t)) - z_2(t))] \\ x_1(\theta) &= \varphi(\theta), \theta \in [-\tau, 0], x_2(0) = x_{20}, z_i(0) = z_{i0}, k_i > 0, h_i > 0, i = 1, 2. \end{aligned} \quad (12)$$

For $p(x) = \frac{1}{x}$ the stationary state of system (12) is given by (4).

With respect to the transformation $u_1(t) = x_1(t) - x_1^*$, $u_2(t) = x_2(t) - x_2^*$, $u_3(t) = z_1(t) - z_1^*$, $u_4(t) = z_2(t) - z_2^*$, and by expanding (12) in a Taylor series around the stationary state $(x_1^*, x_2^*, z_1^*, z_2^*)$ and neglecting the terms of higher order than the first order, we obtain the following linear approximation of system (12) :

$$\begin{aligned}\dot{u}_1(t) &= k_1\{a_{10}u_1(t) + a_{01}u_2(t) + a_{001}u_3(t)\} \\ \dot{u}_2(t) &= k_2\{b_{10}u_1(t - \tau) + b_{01}u_2(t) + b_{001}u_4(t)\} \\ \dot{u}_3(t) &= h_1\{c_{10}u_1(t) + c_{01}u_2(t) + c_{001}u_3(t)\} \\ \dot{u}_4(t) &= h_2\{d_{10}u_1(t) + d_{01}u_2(t) + d_{001}u_4(t)\}\end{aligned}\tag{13}$$

where $a_{10}, a_{01}, a_{001}, b_{10}, b_{01}, b_{001}, c_{10}, c_{01}, c_{001}, d_{10}, d_{01}, d_{001}$, are given by (8).

The corresponding characteristic equation of (13) is :

$$\lambda^4 + n_{43}\lambda^3 + n_{42}\lambda^2 + n_{41}\lambda + n_{40} + e^{-\lambda\tau}(n_{22}\lambda^2 + n_{21}\lambda + n_{20}) = 0,\tag{14}$$

where

$$\begin{aligned}n_{43} &= m_{43}, \quad n_{22} = -k_1k_2a_{01}b_{10} \\ n_{42} &= k_1k_2a_{10}b_{01} + (k_1a_{10} + k_2b_{01})(h_1c_{001} + h_2d_{001}) - h_1k_1a_{001}c_{10} - \\ &\quad - k_2h_2b_{001}d_{01} + h_1h_2c_{001}d_{001} \\ n_{41} &= k_1k_2a_{10}b_{01}(h_1c_{001} - h_2d_{001}) - k_2h_1h_2c_{001}d_{001}b_{01} + \\ &\quad + h_1k_1c_{10}a_{001}(k_2b_{01} + h_2d_{001}) - k_1k_2h_2b_{001}a_{01}d_{10} + \\ &\quad + k_1k_2h_2a_{10}b_{001}d_{01} + k_2h_1h_2b_{001}c_{001}d_{01} \\ n_{40} &= k_1k_2h_1h_2(a_{10}b_{01}c_{001}d_{001} - a_{001}b_{01}c_{10}d_{001} + a_{001}b_{001}c_{10}d_{01} + \\ &\quad + a_{01}b_{001}c_{001}d_{10} - a_{001}b_{001}c_{01}d_{10} - a_{10}b_{001}c_{001}d_{01}) \\ n_{21} &= k_1k_2a_{01}b_{10}(h_2d_{001} + h_1c_{001}) - k_1k_2h_1a_{001}b_{10}c_{01} \\ n_{20} &= k_1k_2h_1h_2(a_{001}b_{10}c_{01}d_{001} - a_{01}b_{10}c_{001}d_{001}).\end{aligned}$$

The roots of (14) depend on τ . Considering τ as parameter, we determine τ_0 so that $\lambda = i\omega$ is a root of (14). Substituting $\lambda = i\omega$ into equation (14) we obtain:

$$\omega^4 - in_{43}\omega^3 - n_{42}\omega^2 + in_{41}\omega + n_{40} + (-n_{22}\omega^2 + in_{21}\omega + n_{20})(\cos\omega\tau - i\sin\omega\tau) = 0.$$

From the above equation we have:

$$\omega^8 + r_6\omega^6 + r_4\omega^4 + r_2\omega^2 + r_0 = 0\tag{15}$$

where

$$\begin{aligned}r_6 &= n_{43}^2 - 2n_{42}, \quad r_4 = n_{42}^2 + 2n_{40} - 2n_{43}n_{41} - n_{22}^2, \\ r_2 &= n_{41}^2 - 2n_{42}n_{40} + 2n_{22}n_{20} - n_{21}^2, \quad r_0 = n_{40}^2 - n_{20}^2.\end{aligned}$$

If ω_0 is a positive root of (15) then there is a Hopf bifurcation and the value of τ_0 is given by:

$$\tau_0 = \frac{1}{\omega_0} \arctg \frac{a_1a_4\omega_0 + a_2a_3}{-a_1a_3 + a_2a_4\omega_0},\tag{16}$$

where $a_1 = \omega_0^4 - n_{42}\omega_0^2 + n_{40}$, $a_2 = -n_{43}\omega_0^3 + n_{41}\omega_0$, $a_3 = n_{22}\omega_0^2 - n_{20}$, $a_4 = n_{21}\omega_0$.

We can conclude with the following theorem:

Theorem 4 (i) If ω_0 is a positive root of (15) and $\text{Re}(\frac{d\lambda}{d\tau})_{\lambda=i\omega_0, \tau=\tau_0} \neq 0$, where τ_0 is given by (16), then a Hopf bifurcation occurs at the stationary state $(x_1^*, x_2^*, z_1^*, z_2^*)$ as τ passes through τ_0 .

(ii) If conditions (11) hold and $n_0 > 0$, then the stationary state is asymptotically stable for any $\tau > 0$.

5 Numerical simulation

For the numerical simulation we use Maple 11 and the following data: $q = 0.3$, $s = 40$, $t_1 = 0.16$, $c_1 = 0.2$, $c_2 = 2$, $k_1 = 0.05$, $k_2 = 0.01$, $h_1 = 0.05$, $h_2 = 0.01$. The stationary state is: $x_1^* = 0.34710$, $x_2^* = 0.0347$, $z_1^* = 0.85075$, $z_2^* = 0.03257$.

For $\tau = 0$ the Routh-Hurwitz conditions are satisfied. Then, the stationary state is stable.

The positive solution of (15) is $\omega_0 = 0.010083$ and $\tau_0 = 164.5979$. For $\tau \in (0, \tau_0)$ the stationary state is asymptotically stable and for $\tau \in [\tau_0, \infty)$ the stationary state is unstable. For $\tau = \tau_0$ there is a Hopf bifurcation.

For $q = 0.3$, $s = 40$, $t_1 = 0.16$, $c_1 = 0.2$, $c_2 = 1.5$, $k_1 = 0.05$, $k_2 = 0.01$, $h_1 = 0.05$, $h_2 = 0.01$. The stationary state is: $x_1^* = 0.4359$, $x_2^* = 0.05813$, $z_1^* = 0.824019$, $z_2^* = 0.059313$.

For $\tau = 0$ the Routh-Hurwitz conditions are satisfied. Then, the stationary state is stable.

The equation (15) has no positive root. Then, the stationary state is asymptotically stable for any $\tau > 0$.

6 Conclusions

In the static model with tax evasion, the parameters q and s characterize the behavior of the firms with respect to evasion. The presented figures allow the analysis of the declared revenues and the profits with respect to s .

For the dynamic model with tax evasion, using Routh-Hurwitz criterion we have determined the conditions for which the stationary state is asymptotically stable.

For the dynamic model with tax evasion and time delay, using the delay τ as a bifurcation parameter we have shown that a Hopf bifurcation occurs when τ passes through a critical value τ_0 .

The direction of the Hopf bifurcation, the stability and the period of the bifurcating periodic solutions will be analyzed in a future paper.

The findings of the present paper can be extended in an oligopoly case.

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